

Name: \_\_\_\_\_

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A2CC Intro to Probability

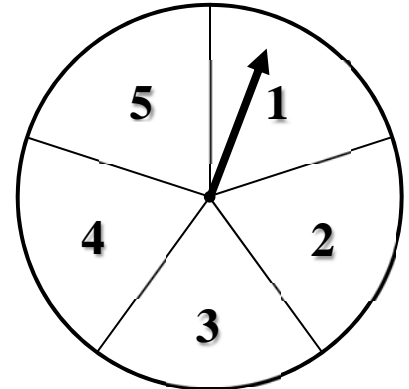
**BASIC PROBABILITY TERMINOLOGY**

1. **Experiment:** Some process that occurs with well defined outcomes.
2. **Outcome:** A result from a single **trial** of the experiment.
3. **Event:** A collection of one or more outcomes.
4. **Sample Space:** A collection of all of the outcomes of an experiment.

**Exercise #1:** An experiment is run whereby a spinner is spun around a circle with 5 equal sectors that have been marked off as shown.

(a) What is the **experiment**?

(b) Give one outcome of the experiment.



(c) What is the probability of spinning the spinner and landing on an odd number? What is the event here? What outcomes fall into the event?

The answer from (c) helps us to define the basic formula that dictates all probability calculations:

**THE BASIC DEFINITION OF PROBABILITY**

The probability of an event  $E$  occurring is given by the ratio:  $P(E) = \frac{n(E)}{n(S)}$ , where:

$n(E)$  is the number of outcomes that fall into the event  $E$

$n(S)$  is the number of outcomes that fall into the sample space

**Exercise #2:** Given the above definition, between what two numbers must ALL probabilities lie? Explain.

When we deal with **theoretical probability** we don't actually have to run the experiment to determine the probability of an event. We simply have to know the number of outcomes in the sample space and the number of outcomes that fall into our event. Let's take a look at a slightly more challenging scenario.

**Exercise #3:** A fair coin is flipped three times and the result is noted each time. The sample space consists of **ordered triples** such as  $(H, H, T)$ , which would represent a head on the first toss, a head on the second toss, and a tail on the third toss.

- (a) Draw a **tree diagram** to show all of the different outcomes in the sample space. (b) List all of the outcomes as ordered triples. How many of them are there?

(c) Find each of the following probabilities based on your answers from (a) and (b):

- (i)  $P(\text{all heads})$  (ii)  $P(\text{exactly 2 heads})$  (iii)  $P(\text{all heads or all tails})$

Sometimes we have to quantify chance by using observations that have been made in the real-world. In this case we talk about **empirical probability**. The fundamental equation for probability still stands.

**Exercise #4:** A survey was done by a marketing company to determine which of three sodas was preferred by people in a blind taste test. The results are shown below.

- (a) Find the empirical probability that a person selected at random from this group would prefer soda B. Express your answer as a fraction and as a decimal accurate to two decimal places (the standard).

Soda	Number who Preferred
A	18
B	24
C	11
Total	53

- (b) Find the empirical probability that a person selected at random from this group would *not* prefer soda A. Again, express your answer as a fraction and as a decimal accurate to two decimal places.

Since the basic calculation within probability involves counting the number of **outcomes** that fit into a particular **event**, it makes sense to have a tool to visualize and keep track of all of the outcomes in a **sample space**. We will do this by using sets. Recall the basic definition of a set:

#### SET DEFINITION

A **set** is simply a collection of things (numbers, objects, etcetera) that satisfy a well-defined criteria. The things that are contained in the set are called the **elements** of the set

**Exercise #1:** The set A is defined as the collection of all integers that are greater than 0 and less than 10.

(a) Write out set A in **roster form**.

(b) Show set A in **Venn Diagram** form. This will be a very simple Venn Diagram.

(c) A **subset** is any set whose elements are all contained within another set. Give two possible rules that could define subsets of A and then write the sets as B and C in roster form. Do sets B and C have any elements in common?

Set B's Definition: \_\_\_\_\_  $B =$

Set C's Definition: \_\_\_\_\_  $C =$

Let's get back to a bit of probability.

**Exercise #2:** Consider an experiment where we first toss a coin and note the outcome and then roll a six-sided die and note the outcome.

(a) Write a set of ordered pairs, such as  $(H, 4)$ , that represents all outcomes for this experiment. Recall that this is called the **sample space**. We will generally call this set S.

(b) Write a set of ordered pairs that represents the event of getting a tail and an even number. Call this set A.

(c) The complement of a set A will be all of the events in the sample space S that do not fall into set A. Write out the complement of set A. We'll call this set B.

(d) Find  $P(A)$  and  $P(B)$ .

A set and its complement are important when we look at probability because all outcomes either fall into an event or into its complement, but not both.

**Exercise #3:** Consider rolling a single six-sided die and recording the result. Let set A be the event of rolling a number greater than 4 and let set B be the complement of set A.

- (a) Draw a Venn Diagram that illustrates the sample space, S, and sets A and B.
- (b) Find  $P(A)$  and  $P(B)$ .
- (c) What is true of the sum  $P(A) + P(B)$ ?
- (d) Prove that the sum of the probability of an event with the probability of its complement will always be 1.

We use the relationship developed in (d) all the time without even thinking about it. Try the following.

**Exercise #4:** Answer each of the following problems by using the relationship developed in Exercise #3(d).

- (a) If the probability I will draw a red marble from a bag is  $\frac{3}{17}$ , what is the probability that I won't draw a red marble from a bag?
- (b) If the probability that it will rain tomorrow is 20%, what is the probability that it won't rain tomorrow?

In theoretical probability calculations, the sets that make up the sample spaces can get difficult to write out. It is good to remember things like tree diagrams to help.

**Exercise #5:** Two four-sided die are rolled and the number on each is noted.

- (a) Draw a tree diagram that represents all outcomes in the sample space. How many are there?
- (b) What is the probability that you don't get two of the same number?

## HOMEWORK

1. Which of the following could *not* be the value of a probability? Explain your choice.

- (1) 53%                      (3)  $\frac{5}{4}$
- (2) 0.78                      (4)  $\frac{3}{4}$
- 

2. If a month is picked at random, which of the following represent the probability its name will begin with the letter J?

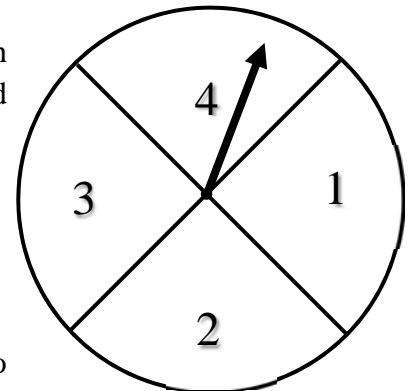
- (1) 0.08                      (3) 0.12
- (2) 0.25                      (4) 0.33
- 

3. If a coin is tossed twice, which of the following gives the probability that it will land both times heads up or both times tails up?

- (1) 0.75                      (3) 0.25
- (2) 0.67                      (4) 0.50
- 

4. A spinner is now created with four equal sized sectors as shown. An experiment is run where the spinner is spun twice and the outcome is recorded each time.

- (a) Create a sample space list of ordered pairs that represent the outcomes, such as  $(4, 2)$ , which represent spinning a 4 on the first spin and a 2 on the second spin.



- (b) Using your answer from (a), determine the probability of obtaining two numbers with a sum of 4.

## APPLICATIONS

5. Samuel pulls two coins out of his pocket randomly without replacement. If his pocket contains one nickel, one dime, and one quarter, what is the probability that he pulled more than 20 cents out of his pocket? Justify your work by creating a tree diagram or a sample space.
6. Janice, Tom, John, and Tamara are trying to decide on who will make dinner and who will wash the dishes afterwards. They randomly pull two names out of a hat to decide, where the first name drawn will make dinner and the second will do the dishes. Determine the probability that the two people pulled will have first names beginning with the same letter. Assume the same person cannot be picked for both.
7. A blood collection agency tests 50 blood samples to see what type they are. Their results are shown in the table below.

(a) If a blood sample is picked at random, what is the probability it will be type B?

(b) If a blood sample is picked at random, what is the probability it will not be type O?

Blood Type	Number of Samples
O	18
A	22
B	7
AB	3
Total	50

(c) Are the two probabilities you calculated in (a) and (b) **theoretical** or **empirical**? Explain your choice.

8. Consider the experiment of picking one of the 12 months at random.

- (a) Write down that sample space,  $S$ , for this experiment. What is the value of  $n(S)$ ? (b) Let  $E$  be the event (set) of picking a month that begins with the letter J. Write out the elements of  $E$ .

- (c) What is the probability of  $E$ , i.e.  $P(E)$ ? (d) What is the probability of picking a month that does *not* start with the letter J?

9. Consider the set,  $A$ , of all integers from 1 to 10 inclusive (that means the 1 and the 10 are included in this set). Give a set  $B$  that is a subset of  $A$ . State its definition and list its elements in roster form. Then give a set  $C$  that is the complement of  $B$ .

Set B's Definition: \_\_\_\_\_

Set B:

Set C:

10. If  $A$  and  $B$  are complements, then which of the following is true about the probability of  $B$  based on the probability of  $A$ ?

- (1)  $P(B) = P(A) + 1$  (3)  $P(B) = \frac{1}{P(A)}$   
(2)  $P(B) = 1 - P(A)$  (4)  $P(B) = P(A) - 1$

11. If a fair coin is flipped three times, the probability it will land heads up all three times is  $\frac{1}{8}$ . Which of the following is the probability that when a coin is flipped three times at least one tail will show up?

- (1)  $\frac{7}{8}$  (3)  $\frac{3}{2}$   
(2)  $\frac{1}{8}$  (4)  $\frac{1}{2}$

12. A four-sided die, in the shape of a tetrahedron, is rolled twice and the number rolled is recorded each time.

(a) Draw a tree-diagram that shows the sample space,  $S$ , of this experiment. How many elements are in  $S$ ?

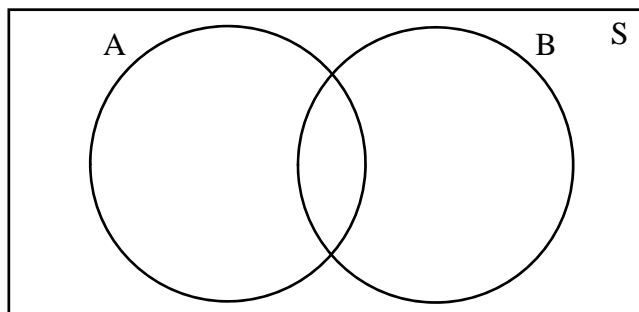
(b) Let  $E$  be the event of rolling two numbers that have an odd product. List all of the elements of  $E$  as ordered pairs.

(c) What is the probability that the two rolled numbers have a product that is odd?

(d) What is the probability that the two rolled numbers have a product that is even?

## REASONING

13. Consider the set of all integers from 1 to 10, i.e.  $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ , to be our sample space,  $S$ . Let  $A$  be the set of all integers in  $S$  that are even and let  $B$  be the set of all integers in  $S$  that are multiples of 3. Fill in the circles of the Venn diagram with elements from  $S$ . If an element lies in both sets, place it in the overlapping region.



14. Find in the following:

$$n(A) =$$

$$n(B) =$$

15. Why is the following equation *not* true? Explain.

$$n(S) = n(A) + n(B)$$